SyDe 312 - Numerical Methods
Unit IV Numerical Integration and Differentiation

Extra quadrature problems

Make sure that you can do Gaussian quadrature problems below by hand if asked, using the standard lookup tables for Gauss-Legendre and Gauss-Laguerre nodes and weights [m-files with these lookup tables are provided]. After you're confident with this, it's ok to use the Matlab *quadl* function to explore practical solution techniques for this type of problem, or the m-files provided that implement Gauss-Legendre and Gauss-Laguerre quadrature.

1. Evaluate the following integral using 2-, 3-, and 6-point Gauss-Legendre quadrature:

$$\int_{-1}^{1} \frac{dx}{x^2\sqrt{x^2+1}}$$

2. Evaluate the following integral using 2-, 3-, and 4-point Gauss-Laguerre quadrature:

$$\int_{0}^{\infty} x^2 e^{-x^2} dx$$

3. Evaluate the integral

$$\int_{1}^{3} \frac{dx}{x^2(100-x^2)^{3/2}}$$

using Gauss-Legendre quadrature with different numbers of points.

4. Using Simpson's 1/3 rule evaluate the integral

$$\int_{2}^{\infty} \frac{dx}{(x-1)^2}$$

and compare the numerical value with the exact answer of 1.

5. Use Matlab to evaluate the complete elliptic integral of the first kind given by

$$K_1(m) = \int_{0}^{\pi/2} \frac{dx}{\sqrt{1-m\sin^2 x}}$$

for $m = 0.5$.

6. Use Matlab to evaluate the complete elliptic integral of the second kind given by

$$K_1(m) = \int_{0}^{\pi/2} \sqrt{1-m\sin^2 x}\, dx$$

for $m = 0.5$.

7. A closed cylindrical barrel of radius R and length L [axis parallel to the ground] is half full with oil of weight density w. The force F exerted by the oil on the circular side is given by

$$F = \int_0^R 2w\sqrt{R^2 - x^2}\, x dx$$

Find the value of F for $R = 1\text{ft}$ and $w = 90\text{lb/ft}^3$ and compare the answers using the following methods:

(a) Symbolic integration (by hand).

(b) Trapezoid rule with 12 steps.

(c) Simpson's 1/3 rule with 12 steps.

(d) Simpson's 3/8 rule with 12 steps.

(e) Gaussian quadrature.

(f) Matlab *quad* and *quadl* functions.